URBAN SCIENCE

Estimation of Curvature: Large Data for Small Problems

6 June 2012

Dr. James G. Wendelberger

Presented at:

QPRC meeting June 4 – 7
at the Long Beach campus of CSU

Summary

- Measurement at the nano-scale (10^{-9}) has resulted in a huge number of measurements over very small regions.
- When determining the curvature of a silicon wafer one may utilize the large number of measurements to estimate curvature.
- How is this best done and how is that related to classical statistical analysis of the curvature estimation problem?

Data Driven

- **University of New Mexico**
- **Micro Electro Mechanical Systems and Nano Electro Mechanical Systems course in NanoScience and MicroSystems Department**
- **Data Analysis - University of New Mexico**
- **Presentation - Urban Science Applications, Inc.**

The Nanoscale

Richard P. Feynman 1959 There's plenty of room at the bottom

- Transcript of a talk given on December 26, 1959, at the annual meeting of ;he American Physical Society at the California Institute of Technology
- JOURNAL OF MICROELECTROMECHANICAL SYSTEMS VOL. **I ,** NO. I . MARCH 1992

"And there is a device on the market, they tell me, by which you can write the Lord's Prayer on the head of a pin. But that's nothing; that's the most primitive, halting step in the direction I intend to discuss. It is a staggeringly small world that is below. In the year 2000, when they look back at this age, they will wonder why it was not until the year 1960 that anybody began seriously to move in this direction."

The Nanoscale

Richard P. Feynman 1959 There's plenty of room at the bottom

- Transcript of a talk given on December 26, 1959, at the annual meeting of ;he American Physical Society at the California Institute of Technology
- JOURNAL OF MICROELECTROMECHANICAL SYSTEMS VOL. I , NO. I . MARCH 1992

"And it turns out that all of the information that man has carefully accumulated in all the books in the world can be written in this form in a cube of material one two-hundredth of an inch wide- which is the barest piece of dust that can be made out by the human eye. So there is plenty of room at the bottom! Don't tell me about microfilm!"

The Nanoscale

Richard P. Feynman 1959 There's plenty of room at the bottom

- Transcript of a talk given on December 26, 1959, at the annual meeting of ;he American Physical Society at the California Institute of Technology
- JOURNAL OF MICROELECTROMECHANICAL SYSTEMS VOL. **I ,** NO. I . MARCH 1992

"MINIATURIZINTG THE COMPUTER

I don't know how to do this on a small scale in a practical way, but I do know that computing machines are very large; they fill rooms. Why can't we make them very small, make them of little wires, little elements-and by little, I mean little. For instance, the wires should be 10 or 100 atoms in diameter, and the circuits should be a few thousand angstroms across."

Roadmap

The Nanoscale

Richard P. Feynman 1959 There's plenty of room at the bottom

- Transcript of a talk given on December 26, 1959, at the annual meeting of ;he American Physical Society at the California Institute of Technology
- JOURNAL OF MICROELECTROMECHANICAL SYSTEMS VOL. **I ,** NO. I . MARCH 1992

- "MINIATURIZATION BY EVAPORATION"
- "PROBLEMS OF LUBRICATION"
- "A HUNDRED TINY HANDS"
- "REARRANGING THE ATOMS"
- "ATOMS IN A SMALL WORLD" – Quantum Effects
- "HIGH SCHOOL COMPETITION"
  - A page down to 1/25,000 scale to be read by an electron microscope - $1,000
  - Operating electric motor in 1/64 inch cube - $1,000

The Nanoscale

Richard P. Feynman 1959 There’s plenty of room at the bottom

- Transcript of a talk given on December 26, 1959, at the annual meeting of ;he American Physical Society at the California Institute of Technology
- JOURNAL OF MICROELECTROMECHANICAL SYSTEMS VOL. **I ,** NO. I . MARCH 1992

“At the atomic level, we have new kinds of forces and new kinds of possibilities, new kinds of effects.”

Table 1: Sensitivity of physical quantities to length scale

Physical Quantities	Examples	Governing Equation	Sensitivity to Length Scale
Van der Waals Forces [2] $\breve{A}$:= *Hamaker Constant* A := *Area* d := *Distance between objects* r := *Radius* r_i := *Radius of object "i"* l := *Facing lengths of two objects*	***Case1:*** Two Spheres	$F = \frac{\breve{A}}{6}\left(\frac{r_1 r_2}{r_1 + r_2}\frac{1}{d^2}\right)$	l^{-1}
	Case 2: A sphere and a surface	$F = \frac{A}{6}\left(\frac{r}{d^2}\right)$	l^{-1}
	Case 3: Two Cylinders	$F = -\frac{A}{8\sqrt{2}}\left(\sqrt{\frac{r_1 r_2}{r_1 + r_2}}\frac{l}{\sqrt{d^5}}\right)$	l^{-1}
	Case 4: Two crossed cylinders	$F = \frac{A}{6}\left(\frac{\sqrt{r_1 r_2}}{d^2}\right)$	l^{-1}
	Case 5: Two Surfaces	$F = \frac{A}{6\pi}\left(\frac{1}{d^3}\right)$	l^{-3}
Viscous forces μ := *Dynamic viscosity* V_0 := *Relative velocity*	***Case 1:*** Two infinite plates	$F = \mu V_0\left(\frac{1}{d}\right)$	l^{-1}
	Case 2: Two finite plates	$F = \mu V_0\left(\frac{A}{d}\right)$	l^{1}
Electrostatic force ϵ_r := *Relative static permittivity* ϵ_0 := *Vacuum permittivity* V_e := *Electrical potential* h := *out of plane thickness*	***Case 1:*** Infinite parallel plates capacitor	$F = \frac{\epsilon_0 \epsilon_r V_e^2}{2}\left(\frac{1}{d^2}\right)$	l^{-2}
	Case 2: Finite parallel plates at distance d	$F = \frac{\epsilon_0 \epsilon_r V_e^2}{2}\left(\frac{A}{d^2}\right)$	l^{0}
	Case 3: Comb drive [3]	$F = \frac{\epsilon_0 \epsilon_r V_e^2}{2}\left(\frac{h}{d}\right)$	l^{0}
Thermal Expansion E_y := *Young modulus of elasticity* α_T := *Therma expansion coefficient* ΔT := *Temperature change*	***Case 1:*** Constrained column	$F = E_y \alpha_T \Delta T A$	l^{2}
Magnetic forces [4] μ_0 := *Vacuum permeability* d := *Distance between wires* l := *Length along wire* I_i := *Current in wire "i"* A_0 := *Cross sectional area* A_s := *Surface area* $\dot{Q}_s$:= *Surface heat flow rate* I_e := *Electrical current*	***Case 1:*** Constant current density with the boundary condition $\frac{I_e}{A_0} = cons.$	$F = \frac{\mu_0}{2\pi}\frac{l}{d} I_1 I_2$	l^{4}
	Case 2: Constant heat flow through the surface of the wire with the boundary condition $\frac{\dot{Q}_s}{A_s} = cons.$	$F = \frac{\mu_0}{2\pi}\frac{l}{d} I_1 I_2$	l^{3}
	Case 3: Constant temperature rise of wire with the boundary condition $\Delta T = cons.$	$F = \frac{\mu_0}{2\pi}\frac{l}{d} I_1 I_2$	l^{2}

Sensitivity of physical quantities to length scale

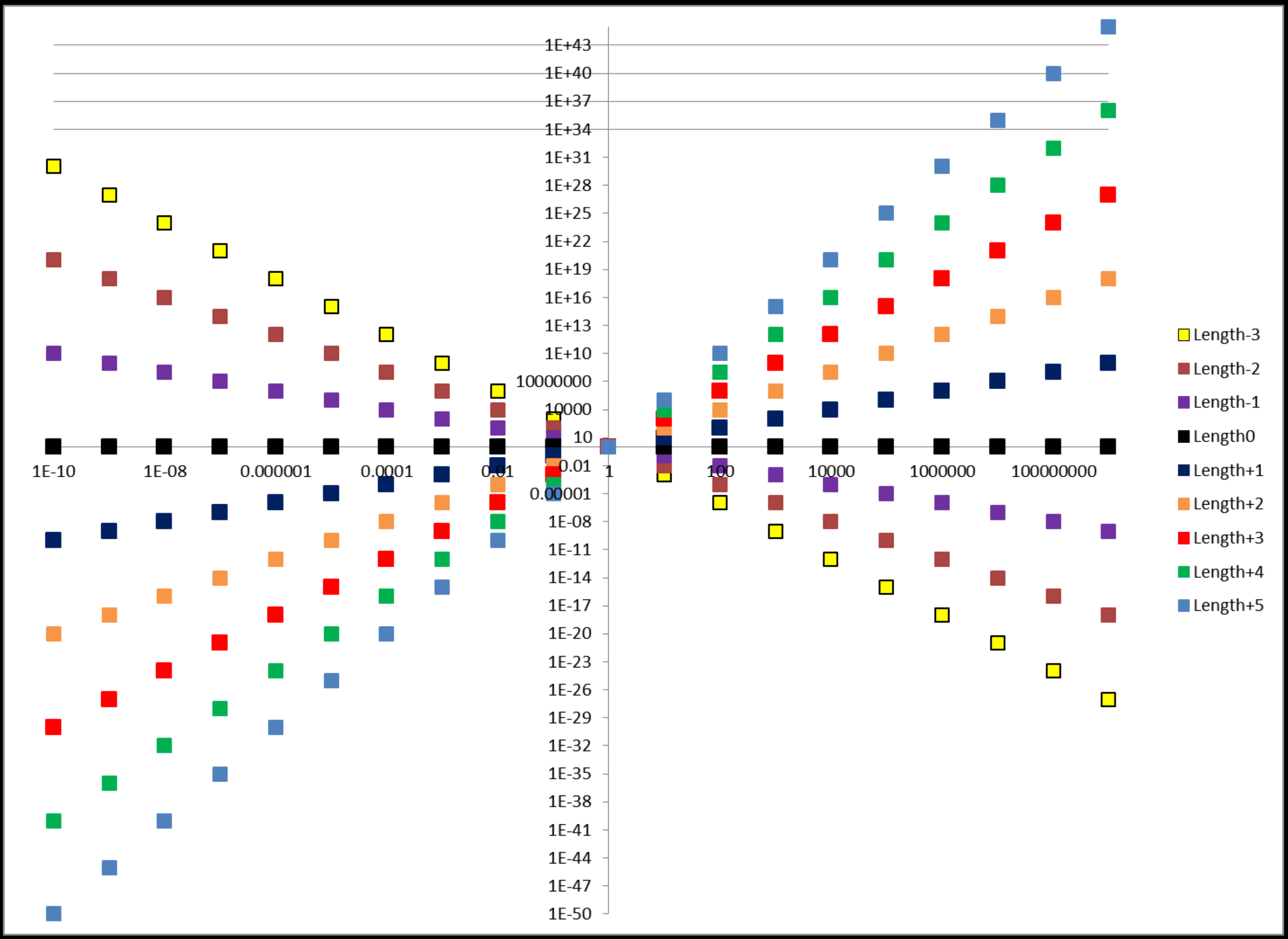
- MEMS and NEMS, UNM NSMS 519, Professor Zayd Leseman

1. Van der Waals – $length^{-3}$ or $length^{-1}$
2. Viscous – $length^{-1}$ or $length^{+1}$
3. Electrostatic – $length^{-2}$ or $length^{0}$
4. Thermal Expansion – $length^{+2}$
5. Magnetic – $length^{+2}$, $length^{+3}$ or $length^{+4}$
6. Piezoelectric – $length^{+1}$ or $length^{+2}$
7. Drag – $length^{+2}$
8. Surface Tension – $length^{+1}$
9. Inertia and Weight – $length^{+3}$
10. Mass Moment of Inertia – $length^{+3}$, $length^{+4}$ or $length^{+5}$
11. Shape memory Alloy – $length^{+2}$

The Forces at the Nanoscale

Sensitivity of physical quantities to length scale

- MEMS and NEMS, UNM NSMS 519, Professor Zayd Leseman



- Van der Waals – length^{-3} or length^{-1}
- Viscous – length^{-1} or length^{+1}
- Electrostatic – length^{-2} or length^{0}
- Thermal Expansion – length^{+2}
- Magnetic – length^{+2}, length^{+3} or length^{+4}
- Piezoelectric – length^{+1} or length^{+2}
- Drag – length^{+2}
- Surface Tension – length^{+1}
- Inertia and Weight – length^{+3}
- Mass Moment of Inertia – length^{+3}, length^{+4} or length^{+5}
- Shape memory Alloy – length^{+2}

The Schrödinger Equation

Dr. Erwin Schrödinger

- Four Lectures on Wave Mechanics.
- Delivered at the Royal Institution, London, on 5, 7 and 14 March 1928

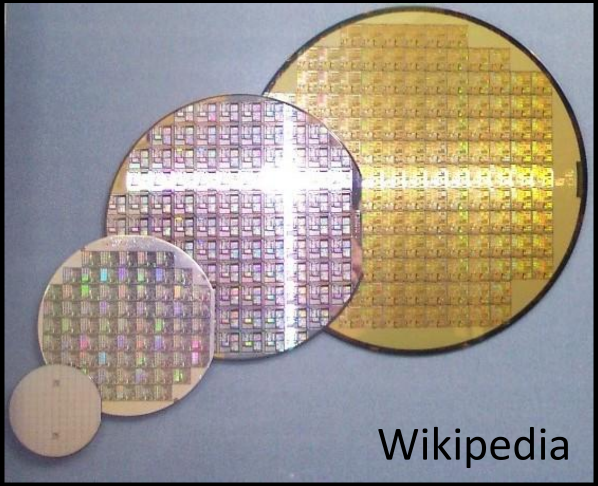
"Substituting from (12) and (8) in (10) and replacing p by Ψ(…) we obtain"

$$\text{“}\nabla^2\Psi + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0\text{”} \quad \text{or} \quad H\Psi = E\Psi$$

"(…) A simplification in the problem of the "mechanical waves" consists in the absence of boundary conditions. I thought the later simplification fatal when I first attacked these equations. Being insufficiently versed in mathematics, I could not imagine how proper vibration frequencies could appear without boundary conditions. Later on I recognized that the more complicated form of the coefficients (i.e. the appearance of V(x,y,z)) takes charge, so to speak, of what is ordinarily brought about by the boundary conditions, namely, the selection of definite values of E."

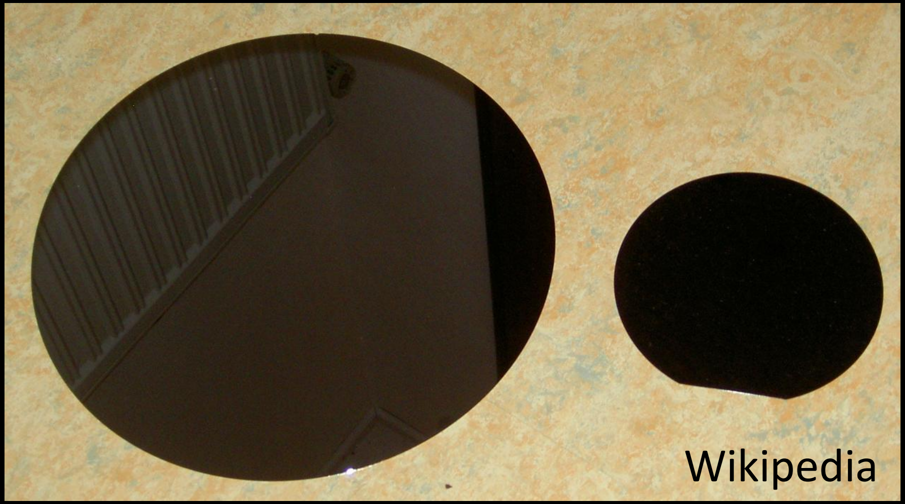
Silicon Wafers

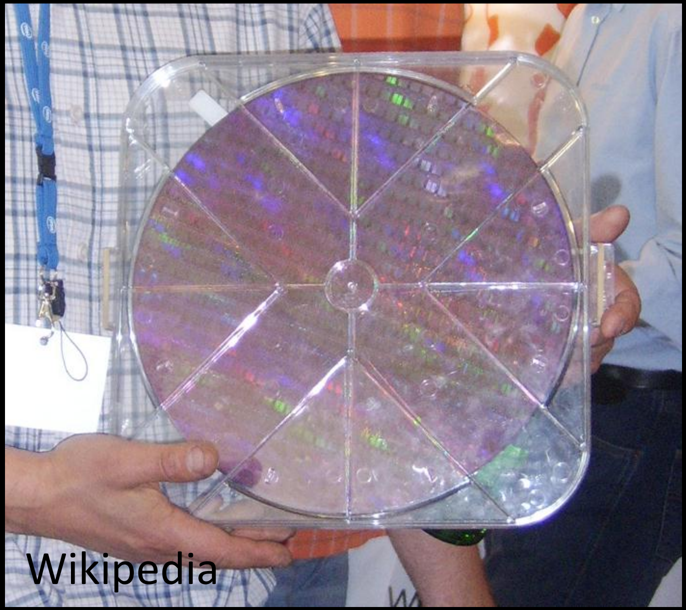
Typical Wafer Sizes: 2, 4, 5, 6, 8, 12 inch



2 inch Wafer

Crystal Orientation





Silicon Wafers

Creation of the Wafer

- Starting Crystal
- Dipping Candles

Single Crystal

Crystal Planes

Glass

Clean Room

Topics for Discussion

The Problem – Measuring Curvature

Available Data – You decide

Data Issues – Zero Gravity?

Analysis – Compare to Classic Methods

Estimates – Fit a Circle

Measuring Curvature

Place on Surface? Van der Waals Force

3 or 4 Point Contact

Leveling

Where to measure?

- Center to Edge
- Edge to Edge

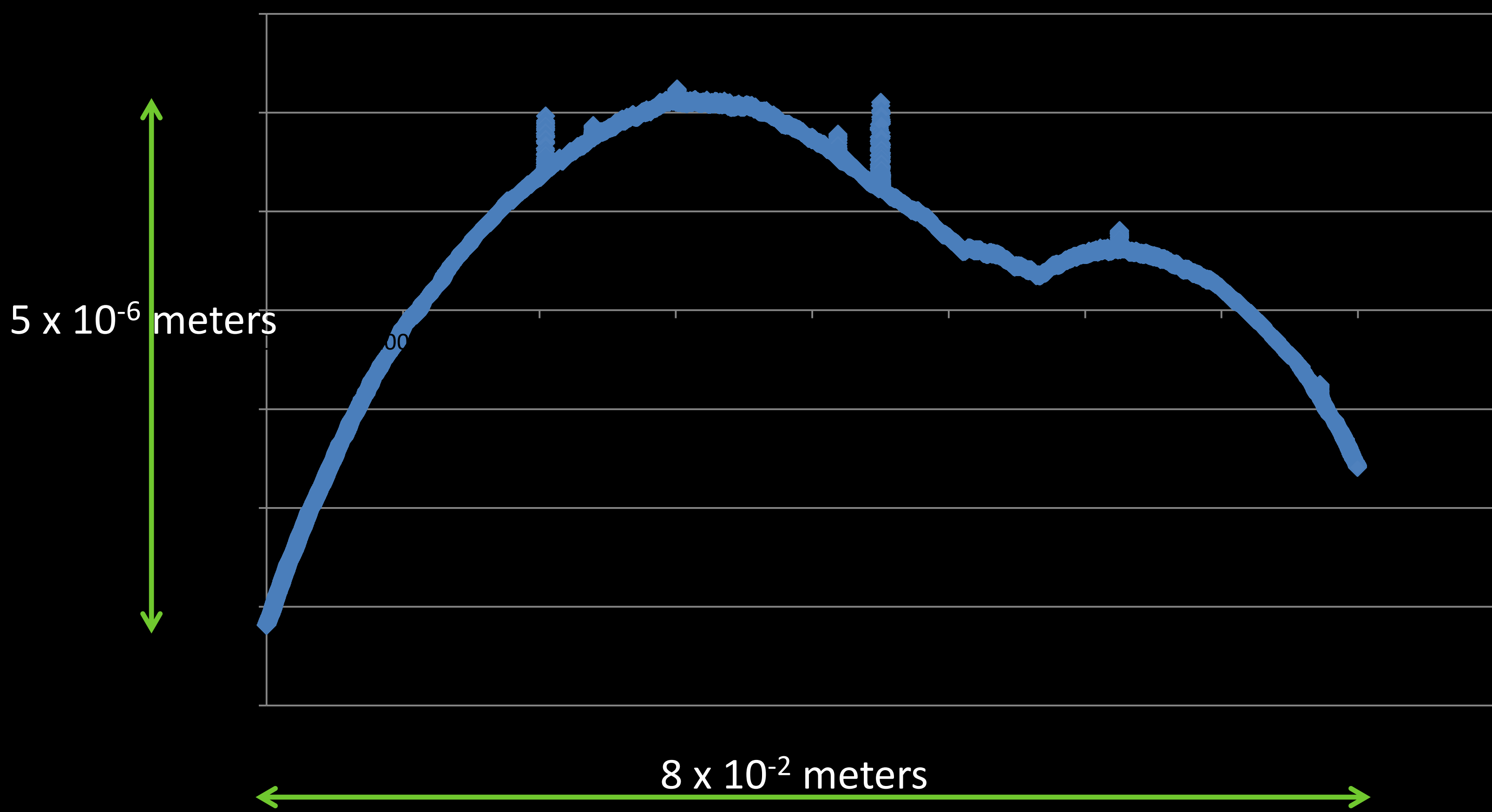
How many measurements?

- 36,000 and 72,000 Points

Measuring Curvature

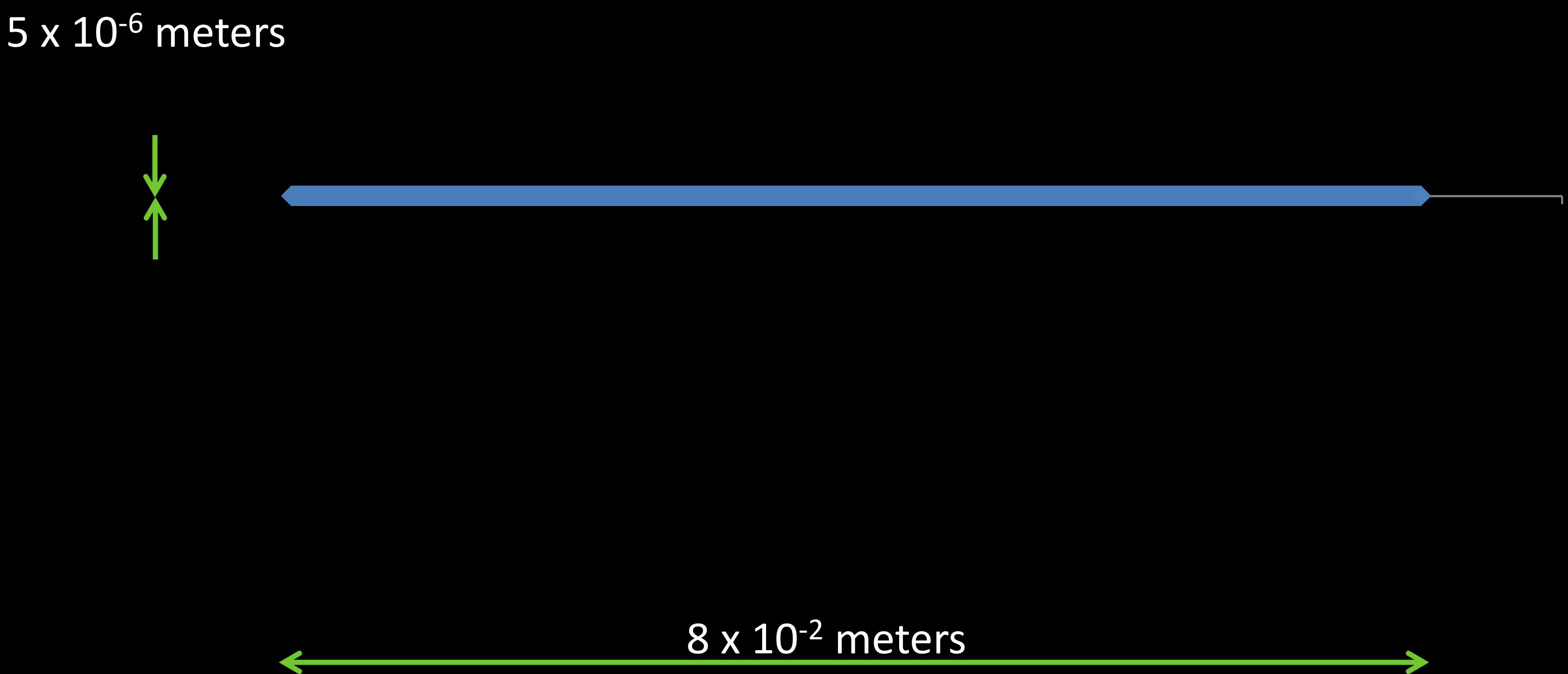
Smoothing the Raw Data

Imperfection – Dust, Ridge Rebound, etc?



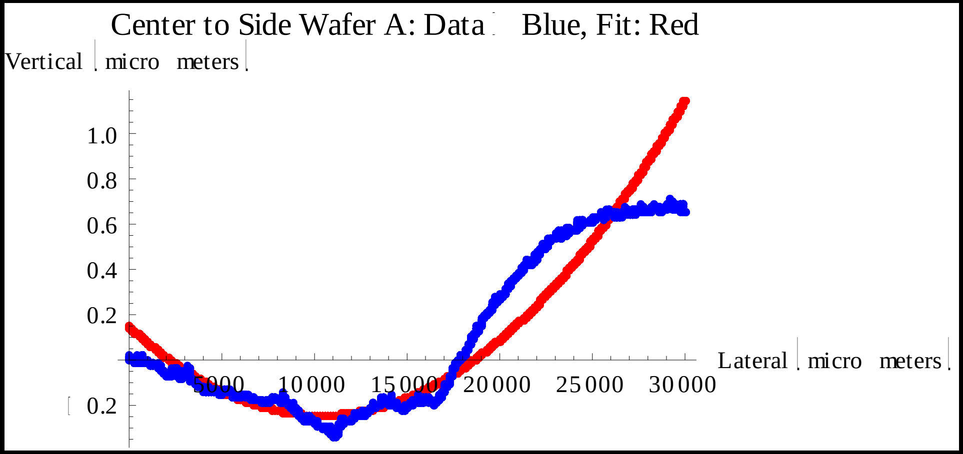
Measuring Curvature

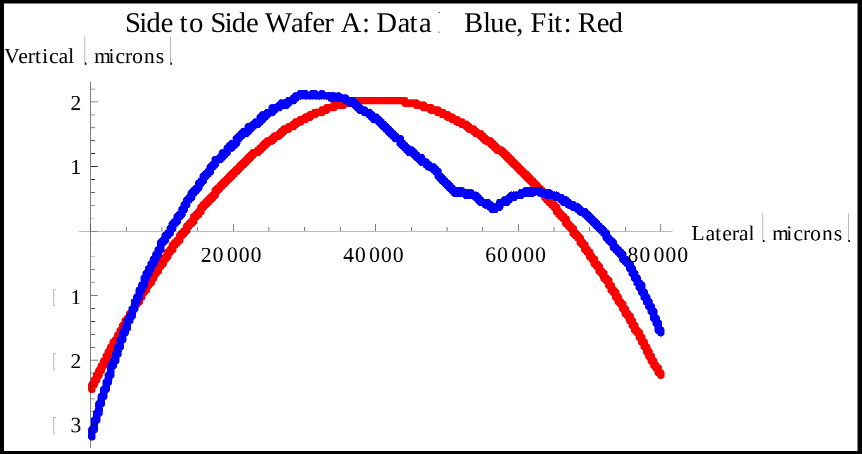
Aspect Ratio of 1:1 Yields A Different Perspective



Measuring Curvature

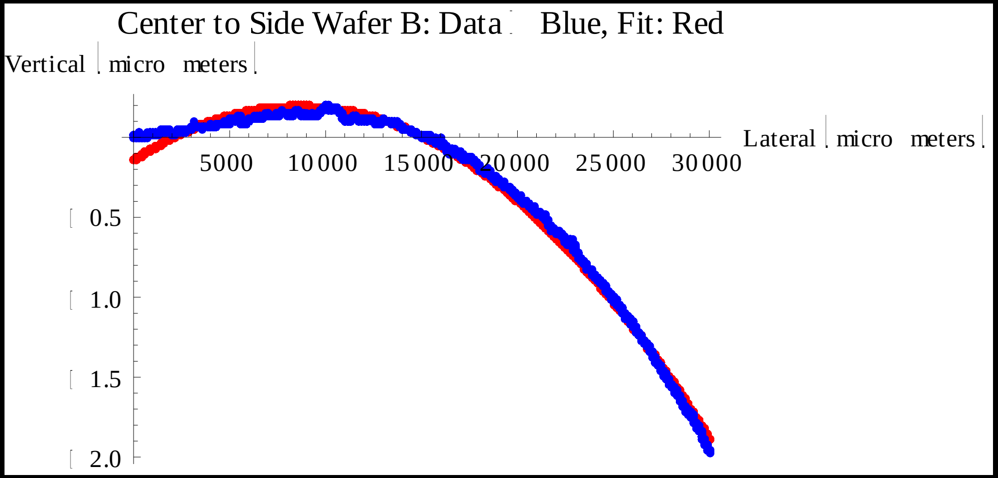
Wafer A, 3 Point Support, Cleaned/Smoothed Data

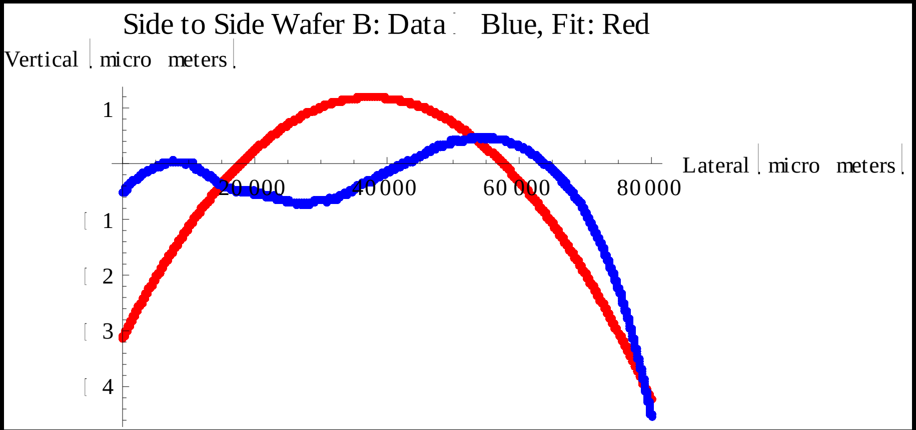




Measuring Curvature

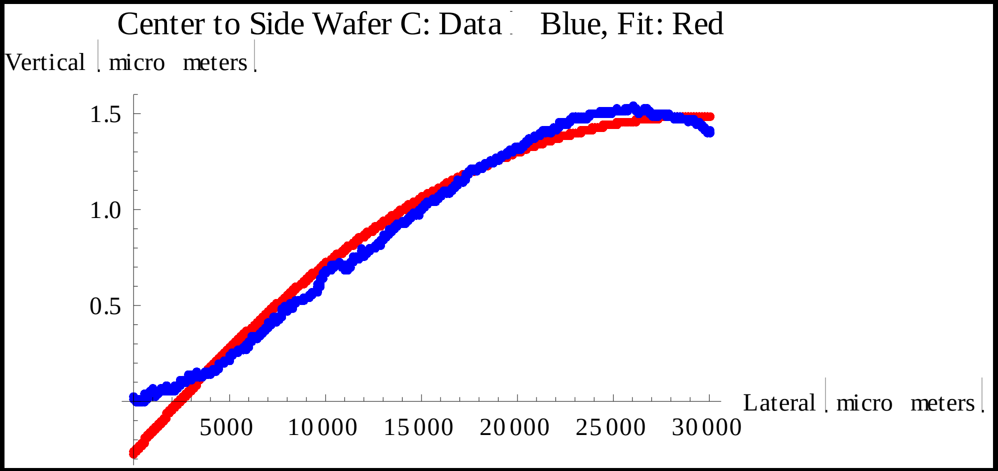
Wafer B, 3 Point Support, Cleaned/Smoothed Data

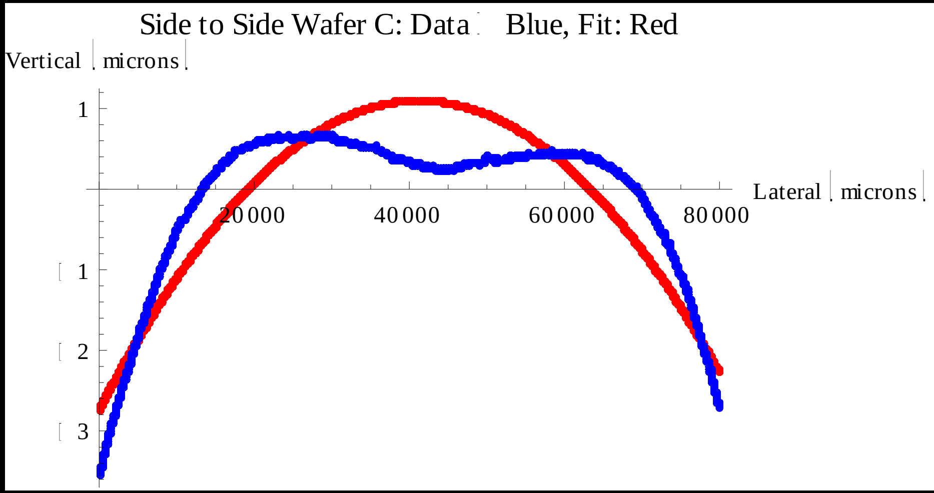




Measuring Curvature

Wafer C, 3 Point Support, Cleaned/Smoothed Data





Measuring Curvature

Linear Scan – Fit a Circle – Radius of Curvature

Random Points - 2 Dimensional Smoothing Spline - Laplacian

Outliers? Do they matter?

Effects of Supports

Proper Force Accounting – Van der Waals, gravity, etc. – Free Space

Measuring Curvature Why?

Strain Causes Curvature – Films or Oxidation

Difference between Data or Curves or Curvature?

Force Modeling

Should we Measure Strain or Stress Directly?

Optimal Design – Random for Now?

Thank you!

Table 1: Sensitivity of physical quantities to length scale

Physical Quantities	Examples	Governing Equation	Sensitivity to Length Scale
Van der Waals Forces [2] $\breve{A}$:= *Hamaker Constant* A := *Area* d := *Distance between objects* r := *Radius* r_i := *Radius of object "i"* l := *Facing lengths of two objects*	***Case1:*** Two Spheres	$F = \frac{\breve{A}}{6}\left(\frac{r_1 r_2}{r_1 + r_2}\frac{1}{d^2}\right)$	l^{-1}
	Case 2: A sphere and a surface	$F = \frac{A}{6}\left(\frac{r}{d^2}\right)$	l^{-1}
	Case 3: Two Cylinders	$F = -\frac{A}{8\sqrt{2}}\left(\sqrt{\frac{r_1 r_2}{r_1 + r_2}}\frac{l}{\sqrt{d^5}}\right)$	l^{-1}
	Case 4: Two crossed cylinders	$F = \frac{A}{6}\left(\frac{\sqrt{r_1 r_2}}{d^2}\right)$	l^{-1}
	Case 5: Two Surfaces	$F = \frac{A}{6\pi}\left(\frac{1}{d^3}\right)$	l^{-3}
Viscous forces μ := *Dynamic viscosity* V_0 := *Relative velocity*	***Case 1:*** Two infinite plates	$F = \mu V_0\left(\frac{1}{d}\right)$	l^{-1}
	Case 2: Two finite plates	$F = \mu V_0\left(\frac{A}{d}\right)$	l^{1}
Electrostatic force ϵ_r := *Relative static permittivity* ϵ_0: =*Vacuum permittivity* V_e := *Electrical potential* h := *out of plane thickness*	***Case 1:*** Infinite parallel plates capacitor	$F = \frac{\epsilon_0 \epsilon_r V_e^2}{2}\left(\frac{1}{d^2}\right)$	l^{-2}
	Case 2: Finite parallel plates at distance d	$F = \frac{\epsilon_0 \epsilon_r V_e^2}{2}\left(\frac{A}{d^2}\right)$	l^{0}
	Case 3: Comb drive [3]	$F = \frac{\epsilon_0 \epsilon_r V_e^2}{2}\left(\frac{h}{d}\right)$	l^{0}
Thermal Expansion E_y := *Young modulus of elasticity* α_T := *Therma expansion coefficient* ΔT := *Temperature change*	***Case 1:*** Constrained column	$F = E_y \alpha_T \Delta T A$	l^{2}
Magnetic forces [4] μ_0 := *Vacuum permeability* d := *Distance between wires* l := *Length along wire* I_i := *Current in wire "i"* A_0 := *Cross sectional area* A_s := *Surface area* $\dot{Q}_s$:= *Surface heat flow rate* I_e := *Electrical current*	***Case 1:*** Constant current density with the boundary condition $\frac{I_e}{A_0} = cons.$	$F = \frac{\mu_0}{2\pi}\frac{l}{d} I_1 I_2$	l^{4}
	Case 2: Constant heat flow through the surface of the wire with the boundary condition $\frac{\dot{Q}_s}{A_s} = cons.$	$F = \frac{\mu_0}{2\pi}\frac{l}{d} I_1 I_2$	l^{3}
	Case 3: Constant temperature rise of wire with the boundary condition $\Delta T = cons.$	$F = \frac{\mu_0}{2\pi}\frac{l}{d} I_1 I_2$	l^{2}

URBAN SCIENCE

Table 1: Sensitivity of physical quantities to length scale

Piezoelectric force [5] ϵ := *Mechanical strain* E_e := *Electrical field* e_p := *Piezoelectric Constant* $\frac{\text{Chanrge density}}{\text{applied strain}}$ E_y^E := *Young modulus at constant* E_e	***Case 1:*** 1-D unconstrained actuation	$F = -e_p V\left(\frac{A_0}{d}\right) + E_y^E \epsilon(A_0)$	l^1 & l^2
Drag Force ρ := *Density* C_d := *Drag Coefficient* A_p := *Projected are normal to flow*	***Case 1:*** Infinite cylinder (C_d =.47) ***Case 2:*** Flat plate perpendicular to flow (C_d = 1.28)	$F = \frac{1}{2}\rho V_0^2 C_d(A_p)$	l^2
Surface Tension Force γ := *Surface Tension* p := *Perimeter*	***Case 1:*** Fluid trapped between two circular plates	$F = \gamma(p)$	l^1
Inertia and Weight g := *Gravity* a := *Acceleration*	***Case 1:*** Weight ***Case 2:*** Inertia Force	$F = \rho g(\forall)$ $F = \rho a(\forall)$	l^3 l^3
$\forall$:= *Volume*			
Mass Moment of Inertia ρ_l := *Mass per unit length* ρ_A := *Mass per unit area* ρ_V := *Mass per unit volume* r := *Radius*	***Case 1:*** Sphere ***Case 2:*** Thin circular disk ***Case 3:*** Slender Bar	$\bar{I} = \frac{8\pi}{15}\rho_V(r^5)$ $\bar{I} = \frac{\pi}{2}\rho_A(r^4)$ $I = \frac{1}{12}\rho_l(l^3)$	l^5 l^4 l^3
Shape-Memory Alloy [6] ν := *Poison's ratio* h_f := *Shape memory alloy film thickness* h_s := *Substrate thickness* R_i := *Initial radius of curvature* R_i := *Radius of curvature of the substrate with SAM film*	***Case 1:*** Force caused by shape memory alloy on a substrate	$F = -\frac{E_y}{1-\nu}\left(\frac{A_0 h_s^2(r_1 - r_2)}{6h_f r_1 r_2}\right)$	l^2